**CHAPTER FIVE**

**5. ELEMENTAARY PROBABILITY**

* 1. **Introduction**
* Probability theory is the foundation upon which the logic of inference is built.
* It helps us to cope up with uncertainty.
* In general, probability is the chance of an outcome of an experiment. It is the measure of how likely an outcome is to occur.

**5.2 Definition of and concepts of some probability terms**

* ***Random (Probability) Experiment:*** It is an experiment that can be ***repeated*** ***any*** number of times under similar conditions and it is possible to enumerate the total number of outcomes without predicting an individual outcome.

***Example 1:*** If a fair die is rolled once it is possible to list all the possible outcomes i.e.

1, 2, 3, 4, 5, 6 but it is not possible to predict which outcome will occur.

***Example 2:*** Tossing a coin two times and observing the ***no of heads*** appearing on the top.

* Outcome: the result of a single trial of a random experiment

***Example 1:*** when a coin is tossed once, then the possible outcomes are: head (H) or tail (T).

***Example 2:*** If the experiment consists of flipping two coins, then the possible outcomes are:

*Outcome* =

* ***Sample space (S):*** is a set of ***all possible*** outcomes of a random experiment and each outcome is called a ***simple event*** (**sample point).**
  + - ***Example 1***: Rolling a die:
    - ***Example 2:*** Tossing a coin once: **.**
    - ***Example 3:*** Tossing a coin twice: **.**
* If a sample space has a finite number of points, it is called a ***finite sample space.*** If it has as many points as there are natural numbers it is called a ***countably infinite sample space***. If it has as manypoints as there are in some interval on the *x* axis, such as it iscalled a ***non countably infinite sample space.*** A sample space that isfinite or countably infinite is often called a *discrete sample space*, whileone that is non countably infinite is called a *non-discrete sample space*.
* ***Event:***Is a ***subset*** of sample space. It is a statement of ***one or more*** outcomes of a random experiment. They are denoted by ***capital*** letters.

***Example:*** Getting an ***odd*** numbers in rolling a die.

***Solution:*** Let is an event of getting ***odd*** numbers. Then

* ***Complement of an event:*** The complement of an event A means ***non- occurrence*** of A and is denoted by which contains those points of the sample space which ***don’t belong*** to A.

***Example:*** a) Find the complement of an event of getting ***odd*** numbers in rolling a die.

b) If tossing two coins and getting ***all heads***.

***Solution:*** a) Let is an event of getting ***odd*** numbers in rolling a die.

b) Let be an event of getting ***all heads*** in tossing two coins

* ***Mutually exclusive (disjoint) events:*** Two events which ***cannot*** happen at the ***same*** time. Or two events Aand *B* are ***mutually exclusive****, or* ***disjoint***, if *A ∩ B* =, that is, if *A* and *B* have no elements in common.

**Example**: Experiment -Toss a coin twice

S= {HH, HT, TH, TT}

Let A= Two heads occur {HH}

B= Two tails occur {TT}

C= At least one head occur {HH, HT, TH}

A ∩ B= φ ⇒ A and B are mutually exclusive events

B ∩ C= φ ⇒ B and C are mutually exclusive events

A ∩ C= {HH} ⇒ A and C are not mutually exclusive

* ***Equally likely events:*** Events that have the ***same*** probability of occurring.
* ***Example:*** when a single die is rolled, each outcome has the same probability of occurrence 1/6.
* **Independent Events**: Two events are independent if the occurrence of one does not

affect the probability of the other occurring.

* **Dependent Events**: Two events are dependent if the first event affects the outcome or occurrence of the second event in a way the probability is changed.
* **Finite sample space**: it is a sample space which consists of a finite number of elements. Suppose that S = {x1, x2, …, xn} where xi’s are possible outcomes of an experiment, then S is a finite sample space.

**5.3 Counting Techniques**

* In order to calculate probabilities, we have to know:
* The number of elements in the ***event*** .
* The number of elements in the ***sample space***..
* That is, in order to judge what is ***probable***, we have to know what is ***possible*.** 
  + In order to determine the ***number*** of outcomes, one can use several rules of counting.
* The addition rule
* The multiplication rule
* The permutation rule
* The combination rule
* To ***list*** the outcomes of the sequence of events, a useful device called ***tree diagram*** is used.
  + - 1. **Multiplication and Addition**

Assume that we have K operations or procedures denoted by O1, O2,…, Ok which can be performed in n1, n2, …., nk ways respectively.

1. **Multiplication Rule:** The number of ways in which O1 then O2 then O3 … and finally Ok can be performed is given by .

* We use the multiplication rule iff the completion of the whole process requires the completion of each and every operation. Meaning, O1 **and** O2 **and**….**and** Ok should all be accomplished. Note that the word “**and”** is very crucial and it is used in the sense of intersection.

***Example1:*** A paint manufacturer wishes to manufacture several different paints. The categories include three types of colors (i.e. red, white, blue), two types of type (i.e. latex and oil) and two types of use (i.e. outdoor & indoor). How many different kinds of paints can be made if a person can ***select*** one color, one type ***and*** one use?

Latex Outdour =RLO

***Solution: Indour*** =RLO

Oil Outdour =ROO

Red Indour =ROI

Latex Outdour =WLO

Indour =WLI

Person White Oil Outdour =WOO 12 different ways.

Indour =WOI

Blue Latex Outdour =BLO

Indour =BLI

Oil Outdour =BOO

Indour =BOI

***Example 2:*** Hawassa University Registrar Office wants to give identity number for students by using 4 digits. The number should be considered by the following numbers only: Hence, how many different ID Numbers could be given by the Registrar ?

a) If repetitions are permitted?

b) If repetitions are not permitted?

***Solution:*** 1st digit 2nd digit 3rd digit 4th digit

a.

b.

**Exercise**

If a test consists of ten multiple choice questions with each permitting four possible answers, how many ways are there in which a student give his answers? Out of these how many ways are there in which all the answers will be correct?

*Remark*: Tree diagram may be used to list all possibilities.

1. **Addition Rule:** Suppose that there are K operations such that no two operations can be performed together. The number of ways in which O1 **or** O2 **or** … **or** Ok can be performed is. Here, unlike the case of multiplication rule, the process can be accomplished if and only if one of the k operations is performed. The k operations are just alternative ways of performing the process. Note that when it comes to the addition rule the word **or** is very crucial.

**Example 1**: There are two transportation means from city A to city B, ***either*** using bus transportation ***or*** train transportation. There are 3 buses and 2 trains. How many ways of transportation is there from city A to city B?

***Solution:*** A person can take any of 5 means of transportation from city A to B.

***Example 2:*** A student goes to the nearest snack (Cafe) to have a breakfast. He can take tea, coffee, ***or*** milk with bread, cake or sandwich. How many possibilities does he have?

***Solution:*** Bread Bread Bread

Tea Cake ***or*** Coffee Cake ***or*** Milk Cake

Sandwich Sandwich Sandwich

***Example 2***: There are two transportation means from city A to city B, ***either*** using bus transportation ***or*** train transportation. There are 3 buses and 2 trains. How many ways of transportation is there from city A to city B?

***Solution:*** A person can take any of 5 means of transportation from city A to B.

* + - 1. Permutation

***Definition:*** ***Permutation*** is an ***arrangement*** of "***n distinct"*** objects in a specific ***order***.

**Permutation Rules:**

**1.** The arrangement of ***n distinct objects*** in a specified order using ***r objects*** at a time is

**2.** The arrangement of n distinct objects ***taken all*** together is or

3. The arrangement of ***n objects*** in which ***are alike*** ***(the same),*** are ***alike (the same)***, ... are ***alike***, then the total number of arrangements is

4. The arrangement of *n distinct* objects ***in a circle*** is **.**

***Example 1: Suppose we have a letters A,B,C,D***

1. How many permutations are there taking all the four?
2. How many permutations are there taken ***two letter*** at a time.?
3. How many different permutation can be made from the letters in the word “CORRECTIO”?

Solutions:

1. Here n=4, there are four distinct object There are 4!=24 permutations.
2. Here
3. Here n=10 of which 2 are C, 2 are O, 2 are R, 1E, 1T, 1I, 1N

k1=2, k2=2, k3=2, k4=k5=k6=k7=1

Using the third rule of permutation, there are 453600 permutations.

**Exercise**

**1.** Six different statistics books, seven different physics books, and 3 different Economics books are arranged on a shelf. How many different arrangements are possible if;

* 1. The books in each particular subject must all stand together
  2. Only the statistics books must stand together

1. Ifthe permutation of the word WHITE is selected at random, how many of the permutations
   * 1. Begins with a consonant?
     2. Ends with a vowel?
     3. Has a consonant and vowels alternating?
2. **Combinations**

* ***Combination*** is a ***selection*** of ***n distinct*** objects ***without*** regard to ***order***.
* It is used when the ***order*** of arrangement is ***not important***, as in the ***selection*** process.
  + The number of combinations of ***r objects*** ***selected*** from ***n objects*** is denoted by

***Example 1:*** Given the letters List the number of permutations & combinations for selecting two letters at a time.

***Solution:***

* ***Note that*** in ***permutation*** AB is ***different*** from BA. But in ***combination*** AB is the ***same*** as BA.

**Example**

1. In how many ways a committee of 5 people be chosen out of 9 people?

Solutions:



1. Among 15 clocks there are two defectives .In how many ways can an inspector chose three of the clocks for inspection so that:
   1. There is no restriction.
   2. None of the defective clock is included.
   3. Only one of the defective clocks is included.
   4. Two of the defective clock is included.

Solutions:



1. If there is no restriction select three clocks from 15 clocks and this can be done in :



1. None of the defective clocks is included.

This is equivalent to zero defective and three non defective, which can be done in:



1. Only one of the defective clocks is included.

This is equivalent to one defective and two non defective, which can be done in:



1. Two of the defective clock is included.

This is equivalent to two defective and one non defective, which can be done in:



* 1. **Approaches to Measuring Probability**
  + There are ***four*** different conceptual approaches to the study of probability theory. These are:

1. The Classical (Mathematical) Approach.
2. The Frequents (Empirical) Approach.
3. Subjective probability
4. Objective probability

1. The Classical (Mathematical) Approach

* + - This approach is used when:
      * All outcomes are ***equally likely***.
      * Total number of outcome is finite, say ***n***.

***Definition****:* If a random experiment with ***"n"*** equally likely outcomes is conducted and out of these ***"k"***outcomes are favorable to an ***event A***, then the probability that an event A occur denoted is defined as:

1. ***Example 1:*** A fair die is tossed once. What is the probability of getting
   1. Number 4?
   2. An odd number?
   3. An even number?
   4. Number 8?

**Solutions:**

First identify the sample space, say S



1. Let A be the event of number 4



1. Let A be the event of odd numbers



1. Let A be the event of even numbers



1. Let A be the event of number 8

 Ø



1. A box of 80 candles consists of 30 defective and 50 non defective candles. If 10 of this candles are selected at random, what is the probability
2. All will be defective.
3. 6 will be non defective
4. All will be non defective

**Solutions**:



1. Let A be the event that all will be defective.



1. Let A be the event that 6 will be non defective.



1. Let A be the event that all will be non defective.



***Exercise:*** If 3 books are picked at random from a shelf containing 5 novels, 3 books of poems and a dictionary, then what is the probability that

a) The dictionary is selected?

b) 2 novels and 1 book of poems are selected?

Limitations:

* It cannot used if the outcomes are not equally likely
* It can be used only on finite sample sample space

### The Frequents Approach (Empirical Probability):

This is based on the relative frequencies of outcomes belonging to an event.

***Definition:*** The probability of an event A is the ***proportion*** of outcomes favorable to A in the ***long run*** when the experiment is repeated under the same condition.

* In a given frequency distribution, the probability of an event A being in a given class is:

***Example***: If records show that 60 out of 100,000 bulbs produced are defective. What is the probability of a newly produced bulb to be defective?

***Solution:*** Let A - be the event that the newly produced bulb is defective.

***Example 2:*** In a sample of 50 people, 22 had type "A", 5 had type "B", 2 had type "AB" and 21 had type "O" blood. Find the probability that a person has blood type "O"?

***Solution:*** Let be the event that a person has blood type "O". Then

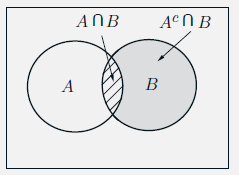
### Axiomatic of Probability

Let **E** be a random experiment and **S** be a sample space associated with **E**. With each event **A** areal number called the satisfies the following properties called ***axioms*** of probability.

1. 2.
2. . 4.
3. if *A* and *B* are ***disjoint (mutually exclusive)*** events, then

***Remark:*** The ***Venn-diagrams*** can be used to solve probability problems.



**Remark**: The axiomatic approach does not give us ways of computing probability but it states certain axioms that a probability of an event should satisfy.

* 1. **Derived theorems of Probability**

## **The addition Rules for Probability**

1. If two events A and B ***are mutually*** exclusive, then the probability that ***A or B*** will occur is

2. If two events A and B are ***not mutually*** exclusive, then the probability that ***A or B*** will occur is .

***Example 1:*** If a ***single card*** is drawn from an ordinary deck and its number is noted, then find the probability that:

a) It is an ***ace* *or* a *diamond***. b) It is an ***ace or a black***. c) It is an ***ace or a Jack***.

***Solution:*** Let ; be the event that an ***ace*** will be selected.

be the event that a ***diamond*** will be selected.

be the event that a ***black*** will be selected.

be the event that a ***jack*** will be selected.

***Example 2:*** Suppose that In adare hospital unit there are 8 nurses and 5 physicians; 7 nurses and 3 physicians are females. If a staff person is selected at random, find the probability that:

a. The subject is a nurse or a male.

b. The subject is a physician or a female.

***Solution:*** The sample space is shown here.

|  |  |  |  |
| --- | --- | --- | --- |
| Staff | Females | Males | Total |
| Nurses | 7 | 1 | 8 |
| Physicians | 3 | 2 | 5 |
| Total | 10 | 3 | 13 |

a.

***Note:*** are any three events, then

* In general, Let be any k events, then

***Exercise:*** The following statement deals with the probability that ***exactly one*** of the events A or B occurs.

Show that .

##### Theorem: If φ is the impossible event then P(φ) = 0

*Proof*: For any event A, A∪ φ =A

P (A∪ φ) = P(A)

P(A) + P(φ) = P(A) since A ∩ φ = φ

P(φ) = 0

***Theorem***: If A/ is the complement of A, then P(A) = 1-P(A/)

*Proof:* S=A∪ A/

P(S) = P(A∪ A/) = 1

P(S) = P(A) + P(A/) since A ∩ A/ = φ

P (A/) = 1- P(A)

***Theorem:*** If A and B are any two events then P (A∪ B) = P (A) + P (B) – P (A ∩ B)

*Proof*: A∪ B = A∪ (A/ ∩ B)

P(A∪ B) = P (A) + P (A/ ∩ B)

And B = (A ∩ B) ∪ (A/ ∩ B)

P(B) = P(A ∩ B) + P(A/ ∩ B)

Thus P(A∪ B) – P (B) = P (A) - P(A ∩ B)

P (A∪ B) = P (A) + P (B) – P (A ∩ B)

***Theorem***: For any events A, B, C

P(A ∪ B ∪ C ) = P(A) + P(B) + P(C) - P(A ∩ B) - P(A ∩ C) - P(B ∩ C) + P(A ∩ B ∩ C)

*Proof*: Let A ∪ B = D

P(D ∪ C) = P (A ∪ B ∪ C )

P (A ∪ B ∪ C ) = P (D) + P (C) - P ((A ∪ B )∩ C)

= P(D) + P(C) – P((A ∩ C) ∪ (B ∩ C))

= P (A ∪ B) + P (C) – P((A ∩ C) ∪ (B ∩ C))

= P(A) + P(B) - P(A ∩ B) + P(C) - P(A ∩ C) - P(B ∩ C)+ P(A ∩ B ∩ C)

P(A ∪ B ∪ C ) = P(A) + P(B) + P(C) - P(A ∩ B) - P(A ∩ C) - P(B ∩ C) + P(A ∩ B ∩ C)

***Theorem***: If A ⊆ B then P(A) ≤ P ( B)

*Proof*: B = A∪ (A/ ∩ B)

P(B) = P (A) + P (A/ ∩ B) – P(A ∩ A/ ∩ B)

P(B) = P (A) + P (A/ ∩ B) since A and A/  are mutually exclusive

Since P (A/ ∩ B) ≥ 0

P(A) ≤ P(B)

***Theorem***: The probability that exactly one of the events A or B occurs is

P(A) + P (B) - 2P (A ∩ B)

*Proof*: P((A ∩ B/) ∪ (B ∩ A/ ))

= P(A ∩ B/) + P (B ∩ A/)

= P (A ∪ B) – P(B) + P(A ∪ B) – P(A)

= 2P(A ∪ B) – P(A) –P(B)

= 2P(A ∪ B) – P(A) –P(B)

= 2P(A) + 2P(B)- 2P(A ∩ B) – P(A) –P(B)

= P(A) + P(B)- 2P(A ∩ B)

###### Examples

* 1. For any events A and B show that P(A ∪ B) ≤ P(A) + P(B)

**Solution**

P (A ∪ B) = P (A) + P (B) – P (A ∩ B)

Since P (A ∩ B) ≥ 0

P (A ∪ B) ≤ P(A) + P(B)

# 5.6 Conditional Probability and Independence

## 1. Conditional Probability

* The conditional probability of an event is a probability obtained with additional information that some other event has already occurred.
* The conditional probability of event ***B*** occurring, ***given that*** event ***A*** has already occurred, can be found by:
* The conditional probability of event ***A*** occurring, ***given that*** event ***B*** has already occurred, can be found by:

***Example 1:*** A card is drawn from an ordinary deck and its number noted. Then it is ***not replaced***. A ***second*** card is drawn and its number noted, and then, find the probability of:

a) Getting ***two Jacks*** (J). b) Getting an ***ace ( A)*** and a ***king (K)*** in order.

c) Getting a ***flower*** and a ***spade***. d) Getting a ***red*** and a ***black*** in order.

***Solution:***  a)

b)

c) .

d)

***Example 2:*** A box contains black chips and white chips. A person selects two chips without replacement. If the probability of selecting a black chip *and* a white chip is and the probability of selecting a black chip on the first draw is , find the probability of selecting the white chip on the second draw, *given* that the first chip selected was a black chip.

**Remark:** (1) 

(2) 

**Remark:** For a given event B, for which P (B)>0, the conditional probability p (A|B) is a legitimate probability law since it satisfies the axioms of probability. That is,

1. 0<P(A|B)
2. P(S|B)=1
3. P (A1) and P(A2) are mutually exclusive events, then

P((A1 UA2 |B= P(A1)

Example 4: To study the proportion of smokers by sex from a population, a random sample of 200 persons was taken. The following table shows the result.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Non-smoker **(A/)** | Smoker **(A)** | Total |
| Female **(B)** | 64 | 16 | 80 |
| Male **(B/)** | 42 | 78 | 120 |
| Total | 106 | 94 | **200** |

Find

1. The probability of getting a non-smoker given that the person selected is a female.
2. The probability of getting a non-smoker given that the person selected is a male.
3. The probability of getting a female given that the person selected is a smoker.

**Solution:**

Let A: A selected person is smoker B: The selected person is female.

a) 

b) 

c) 

**Exercises**

1. Let A and B are two events such that
2. A lot consists of 20 defective and 80 non-defective items from which two items are chosen without replacement. Events A & B are defined as A = {the first item chosen is defective}, B = {the second item chosen is defective}
   1. What is the probability that both items are defective?
   2. What is the probability that the second item is defective?
3. If A and B are two events associated with an experiment and P (A) = 0.5 P (A∪ B) = 0.6 and P (B) = p. Then, find P for which A and B are Independent.
4. A box contains 4 bad and 6 good tubes. Two are drawn together. One of them is tested and found to be good. What is the probability that the other one is also good?
5. If the probability that a student will pass the mid semester exam is 0.7 and the probability that he will pass both the mid and final examinations is 0.4. What is the probability that he will pass the final examination if he passed the mid-semester examination?
6. ***The Multiplication Rules (Theorems)***

The *multiplication rules* can be used to find the probability of two or more events that occur in sequence.

##### Independent events: Two events A and B are independent if the occurrence of "A" does not affect the probability of "B" occurring.

***Dependent events:*** Two events are ***dependent*** if the ***first event affects*** the outcome or occurrence of the ***second*** event in a way the probability is ***changed***.

## 2.1.1 The Multiplication Rules for Probability

1. If two events A and B are ***independent***, then the probability of ***both A and B*** will occur is

2. If two events A and B ***are dependent***, then the probability of ***both A and B*** will occur is

**Note**: Extension of multiplication law of probability for ‘n’ events A1, A2, …, An we have

P (A1 n A2 n…An) = P (A1) P(A2/A1) P (A3/A1 n A2)…P(An/ A1 n A2 n…An-1).

***Example 1:*** A coin is flipped and a die is rolled. Find the probability of getting a head on the coin ***and*** a 4 on the die.

***Solution:*** These two events are *independent* since the outcome of the first event (tossing a coin) does not affect the probability outcome of the second event (rolling a die).

***Example 2:*** An urn contains ***3 red*** balls, ***2 blue*** balls and ***5 white*** balls. A ball is selected and its color noted. Then it is ***replaced***. A ***second*** ball is selected and its color noted. Find the probability of :

a) Selecting two blue balls.

b) Selecting a blue ball and then a white ball.

c) Selecting a red ball and then a blue ball.

***Solution:***  a)

b)

c)

***Exercise :*** If the probabilities are 0.75, 0.7 and 0.525 that a student A, B, or both can solve the problems in a text book respectively. What is the probability that:

a) Student A can only solve the problem selected at random from the book?

b) Student B can only solve the problem selected at random from the book?

***Solution:***

a) b)

a)

## 

## **Total Probability Theorem**

***The law of total probability:*** Suppose are ***disjoint*** events that form *a partition of S(each possible outcome is included in one and only one of the events*   ***)*** *and assume that**p(Bi)>0* such that The probability of an ***arbitrary event A***can be expressed as:

=

* The following Figure illustrates the law for *m* = 5. The event *A* is the ***disjoint union*** of so and for each *i* the multiplication rule states.

B3

A

b1 vBd

S=B

1. ***Bayes’s Theorem***

***Bayes' Rule:*** Suppose the events are ***disjoint (partition of S)*** and The conditional probability of given an ***arbitrary event A***, can be expressed as:

***Example 1:*** ***Box 1*** contains ***2 red*** balls and ***one blue*** ball. ***Box 2*** contains ***3 blue*** balls and ***one red*** ball. A coin is tossed. If it falls ***heads up***, ***Box 1*** is selected and a ball is drawn. If it falls ***tails up***, **Box 2** is selected and a ball is drawn. Then find the probability of selecting a ***red ball***.

***Solution:*** Let; be the event that ***box 1*** is selected.

be the event that ***box 2*** is selected.

be the event that a ***red ball*** is selected.

be the event that a ***blue ball*** is selected

R

B

R

B

* ***R*** is selected, ***if and only if,*** is selected and ***R*** is selected or is selected and ***R*** is selected.

***Example 3:*** A shipment of ***two*** boxes, ***each*** containing ***6*** telephones, is received by a store. ***Box1*** contains ***one defective*** phone and ***box 2*** contains ***2 defective*** phones. After the boxes are ***unpacked (stocked)***, a phone is ***selected*** and ***found to be defective***. Then find the probability that it came from ***box 2***.

***Solution:***  Let be the event that the phone is came from ***box 1***.

be the event that the phone is came from ***box 2***.

be the event that ***defective*** phone is selected.

be the event that ***non defective*** phone is selected.

D

ND

D

ND

* Since,is **selected**, ***iff,*** is **selected** and is **selected** **or**  is **selected** and is **selected**.

1. ***Independent Event***

***Definition (Independent events):*** Two events A and B are said to be independent iff P(A∩B)=P(A) P(B)

Verbally speaking, A and B are independent provided knowledge of the occurrence of A by no means influences the probability of occurrence of B.

**Example**: Let a box contains M red balls and N-M white balls. Let a random sample of 2 balls be drawn successively.

Let A: red ball is drawn in the first draw.

B: red ball is drawn in the second draw.

Are A and B independent?

**Solution**:

Consider the following two cases

1. Sampling with replacement.



1. Sampling with out replacement



Remarks

1. If A and B are mutually exclusive when do they become independent?

P(A ∩ B) = P(φ) ⇒ P(A) P(B) = 0

∴ A and B are independent and mutually exclusive iff A ∩ B = φ and either P(A) =0 or P(B)=0

1. If A ⊆ B then A ∩ B = A

⇒ P(A ∩ B) = P(A)

⇒ A and B cannot be independent unless P(B)= 1

⇒ B = S

1. φ and S are independent of any event

***Theorem***: Let A and B be independent. Then

1. A and B/ are independent
2. A/ and B/ are independent
3. A/ and B are independent

*Remark*: Pair wise independence does not imply independence of several events

**Example**; A box contains four black and six white balls. What is the probability of getting two black balls in drawing one after the other under the following conditions?

1. The first ball drawn is not replaced
2. The first ball drawn is replaced

**Solution**; Let A= first drawn ball is black

B= second drawn is black

Required 

1. 
2. 